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Unitary symmetry and superconvergence relations

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Abstract. We have constructed superconvergent sum rules for the linear combination of invariant amplitudes for meson–baryon elastic scatterings and tested them by saturating with the low-lying intermediate states.

1. Introduction

Consider an amplitude which obeys an unsubtracted dispersion relation and is subject to the asymptotic bound, $|f(\nu)| < \nu^{\alpha}$ for $\nu \to \infty$, where $\nu = (s-u)/4$. If this bound is strong enough so that $\alpha < -1$, we obtain (De Alfaro *et al.* 1966) the superconvergence relation,

$$\int_{-\infty}^{+\infty} \operatorname{Im} f(\nu)\, d\nu = 0. \tag{1}$$

It gives a non-trivial sum rule if the amplitude is odd under crossing. Very few individual amplitudes exist which satisfy all these requirements. Costa and Zimerman (1966) and Seth and Agarwal (1968) have exploited the SU(3) symmetry to construct a superconvergent linear combination of amplitudes for meson–meson scatterings which are separately not superconvergent. The purpose of this note is to use the above method for octet meson–baryon scatterings to get information on some of the decuplet couplings which cannot be measured from decay.

2. Sum rules

The unitary symmetry gives the following relations (Phillips and Rarita 1965) among the invariant amplitudes for meson–baryon elastic scatterings

$$A_\rho(\pi \mathrm{b}) = 2A_\rho(\mathrm{Kb}+\bar{\mathrm{K}}\mathrm{b}), \qquad B_\rho(\pi \mathrm{b}) = 2B_\rho(\mathrm{Kb}+\bar{\mathrm{K}}\mathrm{b}) \tag{2}$$

where the subscript denotes the dominant Regge trajectory in the t channel for the processes indicated in the brackets and b can be N, Σ or Ξ. The meson–lambda scattering does not admit ρ trajectory in the t channel. If we go over to the asymptotic SU(3) limit and carry out the Regge expansions of the invariant amplitudes (Seth *et al.* 1968), we obtain the following high-energy behaviour

$$A_\rho(\pi \mathrm{b}) - 2A_\rho(\mathrm{Kb}+\bar{\mathrm{K}}\mathrm{b}) \simeq \nu^{\alpha_\rho(t=0)-2} \tag{3}$$

$$B_\rho(\pi \mathrm{b}) - 2B_\rho(\mathrm{Kb}+\bar{\mathrm{K}}\mathrm{b}) \simeq \nu^{\alpha_\rho(t=0)-3}. \tag{4}$$

This enables us to write the following non-trivial superconvergence relations

$$\int \operatorname{Im}\{A_\rho(\pi \mathrm{b}) - 2A_\rho(\mathrm{Kb}+\bar{\mathrm{K}}\mathrm{b})\}\, d\nu = 0 \tag{5}$$

$$\int \nu \operatorname{Im}\{B_\rho(\pi \mathrm{b}) - 2B_\rho(\mathrm{Kb}+\bar{\mathrm{K}}\mathrm{b})\}\, d\nu = 0. \tag{6}$$

3. Calculations

We can test the relations (5) and (6) by taking contributions from low-lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ intermediate states in the s channel. The various contributions from the

s-channel Born terms are given below:

$$A^{(1/2)^+} = g^2_{b'bp}\frac{(m_b - m_{b'})}{s - m^2_{b'} + i\epsilon}$$

$$B^{(1/2)^+} = \frac{g^2_{b'bp}}{s - m^2_{b'} + i\epsilon} \tag{7}$$

$$A^{(3/2)^+} = \frac{g^2_{b^*bp}}{3(s - m^2_{b^*} + i\epsilon)}\{2(m_{b^*} + m_b)(E^2_{b^*} - m^2_b) + (s - m^2_b - m^2_p)(E_{b^*} + m_b)\}$$

$$B^{(3/2)^+} = \frac{2g^2_{b^*bp}}{3(s - m^2_{b^*} + i\epsilon)}\{2m^2_b + m_b E_{b^*} - E^2_{b^*}\}$$

with

$$E_{b^*} = \frac{m^2_{b^*} + m^2_b - m^2_p}{2m_{b^*}}.$$

Here b′ denotes $\frac{1}{2}^+$ intermediate baryon, b* denotes $\frac{3}{2}^+$ intermediate baryon resonance and p denotes the pseudoscalar meson π or K.

3.1. *Meson–nucleon scattering*

We take contributions from N, N*(1236) for the πN scattering and from Λ, Σ, Y_1*(1385) for $\bar{\text{K}}$N scattering in the s channel. The t-channel contributions are obtained from (7) by using the appropriate isospin projection operators and crossing matrix. For meson–nucleon scattering the sum rules (5) and (6) become in the narrow-width approximation

$$\frac{2g^2_{N^*N\pi}}{9}\{2(m_{N^*} + m_N)(E^2_{N^*} - m^2_N) + (m^2_{N^*} - m^2_N - m^2_\pi)(E_{N^*} + m_N)\}$$
$$+ 2[(m_N - m_\Lambda)g^2_{\Lambda NK} - (m_N - m_\Sigma)g^2_{\Sigma NK} - \tfrac{1}{3}g^2_{Y_1^*NK}$$
$$\times\{2(m_{Y_1^*} + m_N)(E^2_{Y_1^*} - m^2_N) + (m^2_{Y_1^*} - m^2_N - m^2_K)(E_{Y_1^*} + m_N)\}] = 0 \tag{8}$$

$$\{-2\nu_N g^2_{NN\pi} - \tfrac{4}{9}\nu_{N^*}g^2_{N^*N\pi}(2m^2_N + m_N E_{N^*} - E^2_{N^*})\}$$
$$+ 2\{\nu_N g^2_{\Lambda NK} - \nu_\Sigma g^2_{\Sigma NK} + \tfrac{2}{3}\nu_{Y_1^*}g^2_{Y_1^*NK}(2m^2_N + m_N E_{Y_1^*} - E^2_{Y_1^*})\} = 0 \tag{9}$$

where

$$\nu_{b',b^*} = \frac{m^2_{b',b^*} - m^2_b - m^2_p}{2}.$$

If we use the following SU(3) relations among the couplings

$$g^2_{\Lambda NK} = \tfrac{1}{3}(1 + 2\alpha)^2 g^2_{NN\pi}$$
$$g^2_{\Sigma NK} = (1 - 2\alpha)^2 g^2_{NN\pi}$$

where $\alpha = F/(F+D)$ and $g^2_{NN\pi}/4\pi = 14{\cdot}7$, together with $g^2_{N^*N\pi}/4\pi = 0{\cdot}38/m_\pi{}^2$ from $\Gamma(N^* \to N\pi) = 120$ MeV, we obtain from (8),

$$0{\cdot}45\, g^2_{Y_1^*NK}/4\pi = 4{\cdot}51 - 0{\cdot}86(1 + 2\alpha)^2 + 3{\cdot}67(1 - 2\alpha)^2 \tag{10}$$

and from (9)

$$0{\cdot}47\, g^2_{Y_1^*NK}/4\pi = 2{\cdot}21 - 0{\cdot}29(1+2\alpha)^2 + 2{\cdot}06(1-2\alpha)^2. \qquad (11)$$

The value of α is known (De Swart 1963, SU(6) predicted value of α is 0·4) to lie between 0·25 and 0·4. For $\alpha = 0{\cdot}25$, the value of $m_\pi^2 g^2_{Y_1^*NK}/4\pi$ is 0·15 from (10) and 0·09 from (11). For $\alpha = 0{\cdot}4$ the value of $m_\pi^2 g^2_{Y_1^*NK}/4\pi$ is 0·08 from (10) and 0·06 from (11). These calculated values are consistent with the model-dependent value 0·09 (Dashen *et al.* 1966).

3.2. *Meson–sigma scattering*

We take contributions from Λ, Σ, $Y_1^*(1385)$ for the $\pi\,\Sigma$ scattering and from N, N*(1236), Ξ, $\Xi^*(1530)$ for K Σ scattering in the *s* channel. Using appropriate isospin projection operators and crossing matrix, sum rules (5) and (6) for meson–sigma scattering become in the narrow-width approximation

$$\begin{aligned}
&[-(m_\Sigma - m_\Lambda) g^2_{\Lambda\Sigma\pi} - \tfrac{1}{3} g^2_{Y_1^*\Sigma\pi}\{2(m_{Y_1^*} + m_\Sigma)(E^2_{Y_1^*} - m^2_\Sigma) \\
&\quad + (m^2_{Y_1^*} - m^2_\Sigma - m^2_\pi)(E_{Y_1^*} + m_\Sigma)\}] \\
&\quad + 2[2(m_\Sigma - m_N) g^2_{N\Sigma K} - 2(m_\Sigma - m_\Xi) g^2_{\Xi\Sigma K} \\
&\quad - \tfrac{2}{9} g^2_{N^*\Sigma K}\{2(m_{N^*} + m_\Sigma)(E^2_{N^*} - m^2_\Sigma) + (m^2_{N^*} - m^2_\Sigma - m^2_K)(E_{N^*} + m_\Sigma)\} \\
&\quad - \tfrac{2}{3} g^2_{\Xi^*\Sigma K}\{2(m_{\Xi^*} + m_\Sigma)(E^2_{\Xi^*} - m^2_\Sigma) + (m^2_{\Xi^*} - m^2_\Sigma + m^2_K)(E_{\Xi^*} + m_\Sigma)\}] = 0 \qquad (12)
\end{aligned}$$

$$\begin{aligned}
&\{-\nu_\Lambda g^2_{\Lambda\Sigma\pi} - \nu_\Sigma g^2_{\Sigma\Sigma\pi} + \tfrac{2}{3}\nu_{Y_1^*} g^2_{Y_1^*\Sigma\pi}(2m^2_\Sigma + m_\Sigma E_{Y_1^*} - E^2_{Y_1^*})\} \\
&\quad + 2\{2\nu_N g^2_{N\Sigma K} - 2\nu_\Xi g^2_{\Xi\Sigma K} + \tfrac{4}{9}\nu_{N^*} g^2_{N^*\Sigma K}(2m^2_\Sigma + m_\Sigma E_{N^*} - E^2_{N^*}) \\
&\quad + \tfrac{4}{3}\nu_{\Xi^*} g^2_{\Xi^*\Sigma K}(2m^2_\Sigma + m_\Sigma E_{\Xi^*} - E^2_{\Xi^*})\} = 0. \qquad (13)
\end{aligned}$$

Using the following SU(3) relations among the couplings

$$g^2_{\Lambda\Sigma\pi} = \tfrac{4}{3} g^2_{NN\pi}(1-\alpha)^2, \qquad g^2_{\Sigma\Sigma\pi} = 4\alpha^2 g^2_{NN\pi}$$
$$g^2_{N\Sigma K} = g^2_{NN\pi}(1-2\alpha)^2, \qquad g^2_{\Xi\Sigma K} = g^2_{NN\pi}$$

together with $g^2_{Y_1^*\Sigma\pi}/4\pi = 0{\cdot}11/m_\pi^{\,2}$ from $\Gamma(Y_1^* \to \Sigma\,\pi) = 3{\cdot}6$ MeV, we get

$$\frac{1{\cdot}40\, g^2_{\Xi^*\Sigma K}}{4\pi} - \frac{0{\cdot}62\, g^2_{N^*\Sigma K}}{4\pi} = 4{\cdot}05 + 14{\cdot}7(1-2\alpha)^2 - 1{\cdot}56(1-\alpha)^2 \qquad (14)$$

$$\frac{2{\cdot}61\, g^2_{\Xi^*\Sigma K}}{4\pi} - \frac{0{\cdot}17\, g^2_{N^*\Sigma K}}{4\pi} = -0{\cdot}46 - 1{\cdot}96(1-\alpha)^2 - 0{\cdot}59\alpha^2 + 22{\cdot}93(1-2\alpha)^2. \qquad (15)$$

For $\alpha = 0{\cdot}25$ and 0·4 the right-hand side of equation (14) becomes 6·86 and 4·08, respectively. Using the model-dependent values $g^2_{N^*\Sigma K}/4\pi = 0{\cdot}039/m_\pi^{\,2}$ and $g^2_{\Xi^*\Sigma K}/4\pi = 0{\cdot}091/m_\pi^{\,2}$ (Dashen *et al.* 1966) the left-hand side of equation (14) becomes 5·66. The right-hand side of equation (15) is 4·14 and $-0{\cdot}34$ for the two values of α. However, the model-dependent value for the left-hand side of equation (15) is 11·77. Thus while the sum rule (14) is consistent with the model-dependent calculation, the sum rule (15) is not.

3.3. *Meson–cascade scattering*

We take contributions from Ξ, Ξ*(1530) for πΞ scattering and Λ, Σ, Y_1*(1385), Ω^-(1672) for KΞ scattering in the *s* channel. Using appropriate isospin projection operators and crossing matrix the sum rules (5) and (6) for this case become in the narrow-width approximation

$$[-\tfrac{2}{3}g^2_{\Xi^*\Xi\pi}\{2(m_{\Xi^*}+m_\Xi)(E^2_{\Xi^*}-m^2_\Xi)+(m^2_{\Xi^*}-m^2_\Xi-m^2_\pi)(E_{\Xi^*}+m_\Xi)\}]$$
$$+2[-(m_\Xi-m_\Lambda)g^2_{\Lambda\Xi K}+(m_\Xi-m_\Sigma)g^2_{\Sigma\Xi K}+\tfrac{1}{3}g^2_{Y_1^*\Xi K}\{2(m_{Y_1^*}+m_\Xi)(E^2_{Y_1^*}-m^2_\Xi)$$
$$+(m^2_{Y_1^*}-m^2_\Xi-m^2_K)(E_{Y_1^*}+m_\Xi)\}$$
$$+\tfrac{1}{3}g^2_{\Omega\Xi K}\{2(m_\Omega+m_\Xi)(E^2_\Omega-m^2_\Xi)+(m^2_\Omega-m^2_\Xi-m^2_K)(E_\Omega+m_\Xi)\}]=0 \qquad (16)$$

$$\{-2\nu_\Xi g^2_{\Xi\Xi\pi}+\tfrac{4}{3}\nu_{\Xi^*}g^2_{\Xi^*\Xi\pi}(2m^2_\Xi+m_\Xi E_{\Xi^*}-E^2_{\Xi^*})\}$$
$$+2\{-\nu_\Lambda g^2_{\Lambda\Xi K}+\nu_\Sigma g^2_{\Sigma\Xi K}-\tfrac{2}{3}\nu_{Y_1^*}g^2_{Y_1^*\Xi K}(2m^2_\Xi+m_\Xi E_{Y_1^*}-E^2_{Y_1^*})$$
$$-\tfrac{2}{3}\nu_\Omega g^2_{\Omega\Xi K}(2m^2_\Xi+m_\Xi E_\Omega-E^2_\Omega)\}=0. \qquad (17)$$

Using the following SU(3) relations among the couplings

$$g^2_{\Xi\Xi\pi}=g^2_{NN\pi}(1-2\alpha)^2, \qquad g^2_{\Lambda\Xi K}=\frac{g^2_{NN\pi}}{3}(1-4\alpha)^2$$
$$g^2_{\Sigma\Xi K}=g^2_{NN\pi}$$

together with $g^2_{\Xi^*\Xi\pi}/4\pi = 0{\cdot}076/m_\pi^2$ from $\Gamma(\Xi^*\to\Xi\pi) = 7{\cdot}3$ MeV, we get

$$\frac{1{\cdot}06g^2_{\Omega\Xi K}}{4\pi}-\frac{0{\cdot}88g^2_{Y_1^*\Xi K}}{4\pi}=0{\cdot}67+1{\cdot}96(1-4\alpha)^2 \qquad (18)$$

$$\frac{1{\cdot}95g^2_{\Omega\Xi K}}{4\pi}-\frac{0{\cdot}13g^2_{Y_1^*\Xi K}}{4\pi}=-2{\cdot}95+0{\cdot}30(1-2\alpha)^2+3{\cdot}63(1-4\alpha)^2. \qquad (19)$$

For $\alpha = 0{\cdot}25$ and $0{\cdot}4$ the right-hand side of the equation (18) becomes 0·67 and 1·38, respectively. Using the model-dependent values $g_{\Omega\Xi K}{}^2/4\pi = 0{\cdot}012/m_\pi^2$ and $g^2_{Y_1^*\Xi K}/4\pi = 0{\cdot}013/m_\pi^2$ (Dashen *et al.* 1966) the left-hand side of the equation (18) becomes 0·07. The right-hand side of the equation (19) is −2·87 and −1·64 for the two values of α, while the model-dependent value of the left-hand side is 1·10. Again the sum rule (18) gives more consistent results than the sum rule (19).

4. Discussion

As far as *A* sum rules are concerned we thus find that our values are consistent with those obtained from the model-dependent method of Dashen *et al.* 1966. For νB sum rules the values are not found to be consistent with the model-dependent predictions. This shows that the νB sum rules are not being properly saturated by the low-lying states. Other workers (Graham and Huq 1967, Jones and Scadron 1967) have also found that all sum rules are not saturated on an equal footing.

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